February 1, 2008

LBL-35682
UCB-PTH-94/11

Triviality Bounds in the Next to Minimal Supersymmetric Standard Model *

Yi-Yen Wu

Theoretical Physics Group
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY90-21139.

Abstract

We study the implications of the triviality problem for the Higgs masses and other relevant parameters in the Next to Minimal Supersymmetric Standard Model (NMSSM). By means of triviality, a new way to constrain parameters is proposed, and therefore we are able to derive triviality bounds on the heaviest-Higgs mass, the lightest-Higgs mass, the soft SUSY-breaking parameters, and the vacuum expectation value of the Higgs gauge singlet through a thorough examination of the parameter space. The triviality upper bound on the lightest-Higgs mass predicted by NMSSM is indeed larger than the upper bound predicted by MSSM (the Minimal Supersymmetric Standard Model).

Disclaimer

1 Introduction

There have been many studies about the upper bound on the Higgs mass of the standard model (or its supersymmetric extension) [1, 2, 3]. One of the approaches is based on triviality of the ϕ^4 theory [4]. Due to triviality, the standard model is inconsistent as a fundamental theory but is a reasonable effective theory with momentum cut-off Λ. Furthermore, by requiring that Λ be larger than the Higgs mass in order to maintain the consistency of the standard model as an effective theory, Dashen and Neuberger were the first to derive the triviality upper bound (about 800 GeV) on the Higgs mass in the minimal standard model [4]. Improvements on this triviality upper bound have also been made, including the non-perturbative calculations, or the contributions of gauge couplings and the top Yukawa coupling [5, 6, 7, 8]. So far, supersymmetry is the only viable framework where the Higgs scalar is natural [9, 10]. Therefore, it is important to understand how this triviality upper bound on the Higgs mass behaves in the supersymmetric extension of the standard model, especially the issue of the lightest-Higgs mass.

The Minimal Supersymmetric Standard Model (MSSM) is the most studied supersymmetric extension of the standard model. Another possible extension is the Next to Minimal Supersymmetric Standard Model (NMSSM) with two $SU(2)\times U(1)$ Higgs doublets and one Higgs singlet [10, 11, 12]. The inclusion of the Higgs gauge singlet in NMSSM provides an explanation to the μ problem of MSSM [13]. In addition, the existence of the Higgs singlet is suggested in many superstring models [14, 15] and grand unified supersymmetric models [13]. These features make NMSSM an appealing alternative to MSSM. An important issue about MSSM is the upper bound on the lightest-Higgs mass [16]. Because there is no guarantee that there will be the signal of Higgs particles before we reach the upper bound predicted by MSSM, it is definitely interesting to investigate whether the upper bound on the lightest-Higgs mass predicted by NMSSM can be larger than that predicted by MSSM or not. A pioneering work [12] has been done in this respect.

Similar to the standard model, it is suggested that triviality still persists in NMSSM when the Higgs couplings are strong. In short, triviality means that, given the low-energy values of the Higgs couplings, the Higgs couplings will

eventually blow up at some momentum scale Λ_L (the Landau pole) if it is scaled upward, where Λ_L is determined essentially by the low-energy values of the Higgs couplings. The stronger the low-energy Higgs couplings are, the smaller Λ_L is. This observation certainly implies an upper bound on the Higgs mass. In order to establish an upper bound on the lightest-Higgs mass by means of triviality, one of the possible approaches is to treat NMSSM as an effective theory with momentum cut-off Λ, and then require that the Higgs couplings remain finite beneath the cut-off Λ. We will call this approach the Finite Coupling Constant Formulation (FCCF). In FCCF, based on triviality associated with the RGE's, the upper bounds on the low-energy Higgs couplings can be easily computed once the cut-off Λ is specified. Therefore, the corresponding upper bound on the lightest-Higgs mass can be obtained directly from these triviality upper bounds on the Higgs couplings. The cut-off Λ has to be specified by assuming certain underlying grand unification scheme ($\Lambda = 10^{15} \sim 10^{17}$ GeV in most cases.) There have been several works done in this approach [17, 18, 19, 20]. They all arrived at the same conclusion that the upper bound on the lightest-Higgs mass of NMSSM is indeed larger than that of MSSM. For example, W.T.A. ter Veldhuis [17] reported that the upper bound on the lightest-Higgs mass of NMSSM will be 25 GeV larger than that of MSSM if the top-quark mass is 150 GeV.

However, according to the spirit of the paper by Dashen and Neuberger [4], the approach of FCCF is not sufficient and the result is model-dependent. The formulation of triviality constraints proposed by Dashen and Neuberger is based on the requirement that $\Lambda_L \geq m_{HH}$ (m_{HH}: the heaviest-Higgs mass) in order to ensure the consistency of NMSSM as an effective theory with momentum cut-off $\Lambda \leq \Lambda_L$. We will call this approach the Effective Theory Consistency Formulation (ETCF). In ETCF, the requirement of FCCF is always met because Λ_L is constructed in such a way that Higgs couplings blow up at Λ_L. In this sense, ETCF is stronger and more reasonable than FCCF since ETCF ensures not only the requirement of FCCF but also the consistency of NMSSM. ETCF treats Λ_L as a function of Higgs couplings and the constraint $\Lambda_L \geq m_{HH}$ represents a constraint on the full parameter space, including the Higgs couplings and the soft SUSY-breaking parameters because m_{HH} depends on the full parameter space in general. ETCF does extend the non-trivial implications of triviality

to the full parameter space, whereas the implication of FCCF is limited to the Higgs couplings. Therefore, ETCF is able to constrain every parameter of the full parameter space. These constraints will be computed in Section 4. In addition, there is no need to introduce certain GUT scheme in ETCF because the cut-off Λ is determined dynamically by the triviality constraint through a thorough search of the parameter space. In this sense, the approach of ETCF is more general and model-independent. In conclusion, FCCF is a special case of ETCF. ETCF provides a more reasonable basis for us to extend the triviality constraint to the full parameter space. Since ETCF and FCCF are different in nature, it is worth studying the triviality bounds of NMSSM based on ETCF.

The purpose of this paper is to describe how to establish triviality bounds on the full parameter space based on ETCF. The computation of the NMSSM effective potential in this paper includes the tree-level contributions only. Therefore, all the triviality bounds obtained in Sections 4 and 5 are tree-level results. However, it has been pointed out in several works [17, 18, 20] that the top and stop loop contributions are quite substantial when the stop mass is much larger than the top-quark mass. Hence, the present computations are not very precise. One-loop contributions, including those of the top and stop, must be included in future computations in order to make the predictions of the triviality bounds more precise.

In Section 2, the relevant NMSSM lagrangian and renormalization group equations are given. Triviality is observed in the case of strong Higgs couplings, which implies the Landau-pole behavior of the Higgs couplings. To facilitate the computations of triviality bounds, an analytic expression of the Landau pole Λ_L is also derived. In Section 3, the parametrization of the NMSSM Higgs mass spectrum over the full parameter space is done. The determination of the full parameter space is non-trivial because the minimization of the scalar potential leads to several constraints on the parameters. In Section 4, ETCF is established and the triviality bound is solved through the full parameter space by requiring $\Lambda_L \geq m_{HH}$, where $\tan\beta = 1$ is chosen for the sake of simplicity. Combined with the present experimental lower bound on the Higgs mass, this analysis indicates that a very large portion of the parameter space is excluded. For example, the VEV of the Higgs gauge singlet v_3 can be constrained to: $0.24 M_W \leq |v_3| \leq 0.749 M_W$, where M_W is the mass of W gauge boson. The soft

SUSY-breaking parameters are constrained from above. Furthermore, the above constraints will become stronger if the experimental lower bound on the Higgs mass is raised, which implies a better understanding of the correct parameter ranges. In Section 5, an absolute upper bound of $2.8M_W$ on the lightest-Higgs mass is established by a search through the full parameter space. This absolute upper bound is beyond the reach of LEP.

2 Indication of Triviality in NMSSM

The supersymmetric Higgs scalar potential of the Next to Minimal Supersymmetric Standard Model (NMSSM) at tree level can be written as follows [10, 21]:

$$\begin{aligned} V &= |hN|^2({\Phi^\dagger}_1\Phi_1 + {\Phi^\dagger}_2\Phi_2) + |h{\Phi^\dagger}_1\Phi_2 + \lambda N^2|^2 + \frac{1}{8}g_1^2({\Phi^\dagger}_1\Phi_1 - {\Phi^\dagger}_2\Phi_2)^2 \\ &\quad + \frac{1}{8}g_2^2[({\Phi^\dagger}_1\Phi_1 + {\Phi^\dagger}_2\Phi_2)^2 - 4({\Phi^\dagger}_1\Phi_2)({\Phi^\dagger}_2\Phi_1)] \end{aligned} \tag{1}$$

$\Phi_1 = ({\phi^\dagger}_1, {\phi^0}_1)$ and $\Phi_2 = ({\phi^\dagger}_2, {\phi^0}_2)$ are two SU(2)×U(1) Higgs doublets, and N is a complex singlet. h and λ are the Higgs couplings. It is assumed that only ϕ_1^0, ϕ_2^0 and N acquire non-trivial VEV's v_1, v_2 and v_3 respectively. The scalar-quarks and scalar-leptons do not acquire VEV's, and we can ignore their contributions to the scalar potential when studying the Higgs mass spectrum. Note that the superpotential corresponding to (1) does not contain linear and bilinear terms [10] because these terms lead to naturalness problems. Besides, these terms do not appear in a large class of superstring-inspired models. As for the soft SUSY-breaking terms, a particular V_{soft} is chosen:

$$V_{soft} = m_1^2{\Phi^\dagger}_1\Phi_1 + m_2^2{\Phi^\dagger}_2\Phi_2 - m_{12}^2{\Phi^\dagger}_1\Phi_2 - {m^*}_{12}^2{\Phi^\dagger}_2\Phi_1 \tag{2}$$

in order to have predictive power. However, this particular choice of V_{soft} does not destroy the generality of the conclusions obtained in this paper. The most general V_{soft} will be considered in the last section.

We assume three generations of quarks and leptons together with their supersymmetric partners. As for the renormalization group equations relevant to the Higgs couplings, all the Yukawa couplings are neglected except for the

top Yukawa coupling f_t. The relevant one-loop RGE's of NMSSM are given in [15, 21]:

$$8\pi^2 \frac{d\,g_1^2}{dt} = 11 g_1^4 \tag{3}$$

$$8\pi^2 \frac{d\,g_2^2}{dt} = g_2^4 \tag{4}$$

$$8\pi^2 \frac{d\,g_3^2}{dt} = -3 g_3^4 \tag{5}$$

$$8\pi^2 \frac{d\,f_t^2}{dt} = f_t^2(6f_t^2 + h^2 - \frac{13}{9}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2) \tag{6}$$

$$8\pi^2 \frac{d\,h^2}{dt} = h^2(4h^2 + 2\lambda^2 + 3f_t^2 - g_1^2 - 3g_2^2) \tag{7}$$

$$8\pi^2 \frac{d\,\lambda^2}{dt} = 6\lambda^2(\lambda^2 + h^2) \tag{8}$$

where g_1, g_2 and g_3 are the gauge couplings associated with SU(3), SU(2) and U(1) gauge groups respectively. The parameter t is defined as:

$$\mathrm{t} = \frac{1}{2}\ln(\frac{-q^2}{M_W^2}) \tag{9}$$

where q^2 is the space-like effective square of the momentum at which these couplings are defined. At t=0, the gauge couplings can be determined from the experimentally derived inputs [22]:

$$g_1^2 = 0.126, \;\; g_2^2 = 0.446, \;\; g_3^2 = 1.257 \tag{10}$$

To understand the triviality problem, strong Higgs couplings are assumed, and therefore the top Yukawa coupling f_t and all the gauge couplings can be ignored. This is not an unreasonable assumption considering the following observations: The RGE's (3)-(8) have been studied numerically by Babu and Ma [21]. Their results indicate that gauge couplings are negligible if the ratio $\frac{h^2}{g_1^2} \geq 5$ or $\frac{\lambda^2}{g_1^2} \geq 2$ holds, where $\tan\beta$=1 and the top-quark mass m_t=40$\sim$400 GeV. (See Fig.1 in [21] for a more precise description.) Therefore, it's certainly reasonable to expect that $\frac{h^2}{g_1^2} \geq 5$ or $\frac{\lambda^2}{g_1^2} \geq 2$ holds in the case of strong Higgs couplings. For the sake of self-consistency, the assumption of negligible gauge couplings in the case of strong Higgs couplings will be verified *a posteriori* by the numerical

results of Sections 4 and 5. The assumption of negligible Yukawa coupling f_t is a little obscure. There has been some numerical evidence in the standard model [5] that the determination of triviality bounds on the Higgs mass is insensitive to the top-quark mass m_t if $m_t \leq 200$ GeV, and it may still be true in NMSSM. The assumption of negligible f_t will be checked by the computations of Sections 4 and 5, and it turns out that f_t is important only in certain extreme situations. A detailed discussion will be given in Section 4.

In the case of strong Higgs couplings with negligible f_t and g_i, the one-loop RGE's for the Higgs couplings h and λ are:

$$8\pi^2 \frac{d\,h^2}{dt} = h^2(4h^2 + 2\lambda^2) \tag{11}$$

$$8\pi^2 \frac{d\,\lambda^2}{dt} = 6\lambda^2(\lambda^2 + h^2) \tag{12}$$

There is only one fixed point, the infrared stable fixed point at h^2=0, λ^2=0. Therefore, similar to the Landau pole [3, 10] of the pure ϕ^4 theory, h^2(t) and λ^2(t) diverge at some finite t=t$_L$ (the Landau pole) unless the Higgs couplings vanish. Triviality is clearly indicated at one-loop. Notice that the above conclusion is still valid even if the top Yukawa coupling f_t is included. The general solution of the Landau pole t$_L$ can be found by means of the method of integrating factor [23]:

$$\mathrm{t}_L = \frac{2\pi^2 \lambda_0^{-\frac{2}{3}} \sqrt{1 + C\lambda_0^{-\frac{4}{3}}}}{C} - \frac{2\pi^2 \ln(\sqrt{1 + C\lambda_0^{-\frac{4}{3}}} + \sqrt{C\lambda_0^{-\frac{4}{3}}}\,)}{C^{\frac{3}{2}}} \tag{13}$$

$$C = h_0^4\,\lambda_0^{-\frac{8}{3}} + 2\,h_0^2\,\lambda_0^{-\frac{2}{3}} \tag{14}$$

$$h_0 = h(\mathrm{t}=0), \quad \lambda_0 = \lambda(\mathrm{t}=0), \quad \Lambda_L \equiv M_W \cdot \exp(\mathrm{t}_L) \tag{15}$$

where Λ_L is the momentum corresponding to the Landau pole t$_L$. (13)-(15) will be useful to the computations of triviality bounds in Section 4.

Notice that the treatment of triviality here is of perturbative nature. Although the problem of triviality should be of non-perturbative nature, several non-perturbative numerical simulations have been performed [24, 25] and the results indicated that the renormalized perturbative calculation gives essentially the correct triviality upper bound on the Higgs mass. This observation may justify our approach as a first approximation.

3 Parameter Space and Higgs Mass Spectrum

Consider the tree-level scalar potential $V'=V+V_{soft}$, and the relevant parameters are h, λ, v_1, v_2, v_3, m_1^2, m_2^2, m_{12}^2. Without loss of generality, our convention is to take h, λ, m_1^2, m_2^2 to be real, and v_1, v_2, v_3, m_{12}^2 to be complex, i.e., $v_1 = \tilde{v}_1 \, e^{i\phi_1}$, $v_2 = \tilde{v}_2 \, e^{i\phi_2}$, $v_3 = \tilde{v}_3 \, e^{i\phi_3}$, $m_{12}^2 = \tilde{m}_{12}^2 \, e^{i\phi_m}$. The minimization of the scalar potential V' leads to three complex vacuum constraints on these parameters:

$$h^2(|v_1|^2 + |v_2|^2)v_3 + 2\lambda(h v_1^* v_2 + \lambda v_3^2)v_3^* \;=\; 0 \qquad (16)$$

$$\frac{1}{4}(g_1^2+g_2^2)(|v_1|^2-|v_2|^2) + h^2(|v_2|^2+|v_3|^2) + m_1^2 \;=\; \frac{v_2}{v_1}(m_{12}^2 - h\lambda v^{*2}_3) \qquad (17)$$

$$\frac{1}{4}(g_1^2+g_2^2)(|v_2|^2-|v_1|^2) + h^2(|v_1|^2+|v_3|^2) + m_2^2 \;=\; \frac{v_1}{v_2}(m^{*2}_{12} - h\lambda v_3^2) \qquad (18)$$

In addition, one has the following physical constraint:

$$M_W^2 \;=\; \frac{1}{2}g_2^2(|v_1|^2+|v_2|^2) \qquad (19)$$

Imaginary parts of the constraints (16)-(18) fix the phases among the complex parameters v_1, v_2, v_3, m_{12}^2, and (19) reduces ($\tilde{v}_1$, $\tilde{v}_2$) to a single parameter $\tan\beta = \frac{\tilde{v}_2}{\tilde{v}_1}$. Furthermore, ($h$, λ, $\tilde{v}_3$) can be expressed in terms of other parameters by means of the real parts of the constraints (16)-(18). The parameter space under study is then defined as the set of parameters (ϕ, $\tan\beta$, m_1^2, m_2^2, $\tilde{m}_{12}^2$), where

$$0 \;\leq\; \phi \;<\; 4\pi, \;\; -\infty \;<\; \tan\beta,\; m_1^2,\; m_2^2,\; \tilde{m}_{12}^2 \;<\; \infty \qquad (20)$$

The other parameters can be expressed in terms of (20):

$$\tilde{v}_3 \;=\; \frac{M_W}{g_2}\sqrt{\frac{2\,A\,B\,\tan\beta - B^2(1+\tan^2\beta)}{A^2(1+\tan^2\beta)}}$$

$$A \;=\; \tilde{m}_{12}^2(\tan\beta - \frac{1}{\tan\beta}) - m_1^2 + m_2^2 + M_W^2(1+\frac{g_1^2}{g_2^2})\frac{\tan^2\beta - 1}{\tan^2\beta+1}$$

$$B \;=\; \frac{1}{\tan\beta}\{m_1^2 + \frac{1}{2}(1+\frac{g_1^2}{g_2^2})M_W^2\} - \tan\beta\{m_2^2 + \frac{1}{2}(1+\frac{g_1^2}{g_2^2})M_W^2\} \qquad (21)$$

$$v_1 \;=\; \frac{M_W}{g_2}\frac{\sqrt{2}}{\sqrt{\tan^2\beta+1}}, \;\; v_2 = \frac{M_W}{g_2}\frac{\sqrt{2}\,\tan\beta}{\sqrt{\tan^2\beta+1}}\,e^{i\phi}, \;\; v_3 = \tilde{v}_3\,e^{i\frac{\phi}{2}} \qquad (22)$$

$$m_{12}^2 \;=\; \tilde{m}_{12}^2\, e^{-i\phi} \tag{23}$$

$$h^2 \;=\; \frac{\tilde{m}_{12}^2\,\tan\beta - m_1^2 + \frac{M_W^2}{2}(1+\frac{g_1^2}{g_2^2})\frac{\tan^2\beta-1}{\tan^2\beta+1}}{\tilde{v}_3^2 + \tan\beta\,\frac{M_W^2}{g_2^2}(\frac{\tan\beta}{\tan^2\beta+1}\;\pm\;\sqrt{\frac{\tan^2\beta}{(\tan^2\beta+1)^2}\;-\;\frac{g_2^2\,\tilde{v}_3^2}{M_W^2}}\;)} \tag{24}$$

$$\lambda \;=\; h\,\frac{M_W^2}{g_2^2\,\tilde{v}_3^2}(-\frac{\tan\beta}{\tan^2\beta+1}\;\pm\;\sqrt{\frac{\tan^2\beta}{(\tan^2\beta+1)^2}\;-\;\frac{g_2^2\,\tilde{v}_3^2}{M_W^2}}\;) \tag{25}$$

$$\tilde{v}_3,\; h,\; \text{and } \lambda \text{ are real.} \tag{26}$$

$''\pm''$ in (24) and (25) indicates $(h,\ \lambda)$ has two solutions, where the solution with $''+''$ is denoted as $(h_A,\ \lambda_A)$ and the solution with $''-''$ is denoted as $(h_B,\ \lambda_B)$.

(20)-(26) specify the full parameter space. (26) is non-trivial because square roots are involved in (21), (24) and (25). Together with (25), the reality condition of λ leads to:

$$\tilde{v}_3 \;\leq\; \frac{|\tan\beta|}{g_2(\tan^2\beta+1)}M_W \tag{27}$$

$\frac{|\tan\beta|}{tan^2\beta+1}$ has its maximum$=\frac{1}{2}$ at $\tan\beta=\pm 1$. Therefore, we are able to establish an absolute upper bound on $\tilde{v}_3$:

$$\tilde{v}_3 \;\leq\; \frac{1}{2\,g_2}M_W \tag{28}$$

Given $M_W = 80\,\mathrm{GeV}$ and $g_2^2 = 0.446$ from (10), this absolute upper bound on the magnitude of v_3 is 60 GeV. As $\tilde{v}_3 \to \frac{M_W}{2g_2}$, (27) implies $|\tan\beta| \to 1$. Therefore, in NMSSM, $\tan\beta$ can be constrained by means of $|v_3|$, and vice versa.

For the sake of simplicity, we choose $\tan\beta = 1$ when studying (20)-(26). When $\tan\beta = 1$, $\tilde{v}_3$ becomes a free parameter, and $m_1^2 = m_2^2$ is required by the minimization of the scalar potential V': (17) and (18). On the whole, the number of free parameters is unchanged. The full parameter space ($\tan\beta = 1$) is then defined as the set of parameters ($\phi,\ \tilde{v}_3,\ m_1^2 = m_2^2,\ \tilde{m}_{12}^2$) plus the following constraints:

$$0 \;\leq\; \phi \;<\; 4\pi,\;\; 0 \;<\; \tilde{v}_3 \;\leq\; \frac{1}{2\,g_2}M_W \tag{29}$$

$$-\infty \;<\; m_1^2 = m_2^2 \;<\; \tilde{m}_{12}^2 \;<\; \infty \tag{30}$$

There is no essential change to the expressions of the other parameters except

for h and λ:

$$h^2 = \frac{\tilde{m}_{12}^2 - m_1^2}{\tilde{v}_3^2 + \frac{M_W^2}{g_2^2}(\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{g_2^2\,\tilde{v}_3^2}{M_W^2}})} \tag{31}$$

$$\lambda = h\,\frac{M_W^2}{g_2^2\,\tilde{v}_3^2}(-\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{g_2^2\tilde{v}_3^2}{M_W^2}}) \tag{32}$$

(29)-(32) specify the full parameter space and will be studied later. Notice that (31) and (32) imply:

$$h^2,\ \lambda^2 \ \propto\ (\tilde{m}_{12}^2 - m_1^2) \tag{33}$$

and $m_1^2 = m_2^2 < \tilde{m}_{12}^2$ in (30) is the consequence of the reality condition of h.

Based on the parameter space described in (20) or (29), it is trivial to work out the Higgs squared-mass spectrum from the tree-level potential V'. In general, $\{\phi_1^0,\ \phi_2^0,\ N\}$ does not mix with $\{\phi_1^\dagger,\ \phi_2^\dagger\}$. The squared-mass matrix $[M_n^2]$ for $\{\phi_1^0,\ \phi_2^0,\ N\}$ is a 6×6 matrix, and the squared-mass matrix $[M_c^2]$ for $\{\phi_1^\dagger,\ \phi_2^\dagger\}$ is a 4×4 matrix. As expected, $[M_n^2]$ contains five neutral Higgs bosons and one massless particle. $[M_c^2]$ contains two charged Higgs bosons and two massless particles. The detailed expressions of $[M_n^2]$ and $[M_c^2]$ have been given in [20] and won't be repeated here. However, there are several symmetries of $[M_n^2]$ and $[M_c^2]$:

$$[M_n^2] \text{ and } [M_c^2] \text{ are periodic in } \phi, \text{ of periodicity } \pi. \tag{34}$$

In addition, $[M_n^2]$ and $[M_c^2]$ are invariant under two discrete symmetries on the parameter space:

$$h \to -h,\ \ \lambda \to \lambda,\ \ \tan\beta \to -\tan\beta,\ \ m_{12}^2 \to -m_{12}^2 \tag{35}$$

$$h \to -h,\ \ \lambda \to -\lambda \tag{36}$$

As for the Higgs squared-mass spectrum, we are interested in these two quantities: m_{HH} (the heaviest-Higgs mass) and m_{LH} (the lightest-Higgs mass). In Sections 4 and 5, the triviality bounds B_{HH} (the upper bound on m_{HH}) and B_{LH} (the upper bound on m_{LH}) will be derived. In general, m_{HH} and m_{LH} have to be computed from $[M_n^2]$ numerically, and there are no simple analytic expressions. However, the understanding of the dependence of the lightest-Higgs

mass m_{LH} on the Higgs coupling h will be very useful in establishing the triviality bounds. Using the fact that the smallest eigenvalue is smaller than the smallest diagonal term and choosing an appropriate basis for $[M_n^2]$, Binétruy and Savoy [19] have derived an upper bound on m_{LH} ($\tan\beta = 1$):

$$B_{LH}^{(BS)} = \frac{h}{\sqrt{2}} v \geq m_{LH}, \qquad v \approx 250\,\mathrm{GeV} \tag{37}$$

Notice that (37) is the result of tree-level computations. This upper bound is denoted as $B_{LH}^{(BS)}$ in order to be distinguished from B_{LH} (the triviality upper bound on m_{LH}). Therefore, the dependence of m_{LH} on h can be understood as: $m_{LH} \leq B_{LH}^{(BS)} \propto h$, where $B_{LH}^{(BS)}$ is proportional to the Higgs coupling h. Combined with the present experimental lower bound on the Higgs mass, $m_{LH} \leq B_{LH}^{(BS)} = \frac{h}{\sqrt{2}} v$ implies that h must be bounded from below. Notice that $B_{LH}^{(BS)}$ is introduced for illustrative purpose only. In practice, the precise determination of this lower bound on h is made by solving m_{LH} from $[M_n]$ and requiring $m_{LH} \geq$ the experimental lower bound. By means of triviality, it will be argued in Section 4 that h decreases with the soft SUSY-breaking parameter m_1, which enables us to establish an upper bound on the soft SUSY-breaking parameter m_1 based on the lower bound on h. Details will be given in Section 4.

4 Constraints on the Higgs Mass and the Soft SUSY-Breaking Parameters

Based on the parameter space ($\phi,\ \tilde{v}_3,\ m_1^2 = m_2^2,\ \tilde{m}_{12}^2$) specified by (29)-(32), $\Lambda_L \geq m_{HH}$ (m_{HH}: the heaviest-Higgs mass) is required by ETCF, and B_{HH} (the triviality upper bound on the heaviest-Higgs mass m_{HH}) is established by $\Lambda_L = m_{HH} \equiv B_{HH}$, where Λ_L is defined in (13)-(15) and m_{HH} is computed from $[M_n^2]$ numerically. Geometrically, the triviality upper bound B_{HH} defines a surface in the parameter space by means of $\Lambda_L = m_{HH}$, and our convention is to parametrize this triviality surface in terms of ($\phi,\ \tilde{v}_3,\ m_1^2 = m_2^2$), where $\tilde{m}_{12}^2$ depends on ($\phi,\ \tilde{v}_3,\ m_1^2 = m_2^2$) through $\Lambda_L = m_{HH}$. At any point ($\phi,\ \tilde{v}_3,\ m_1^2 = m_2^2$) of this triviality surface, the seven non-zero eigenvalues of $[M_n]$ and $[M_c]$ define the seven triviality upper bounds on the seven physical Higgs masses respectively. For example, the triviality upper bound on m_{LH} (m_{LH}: the lightest-Higgs

mass) is defined as the m_{LH} evaluated on the triviality surface. Therefore, care should be taken in distinguishing m_{LH} from the triviality upper bound on m_{LH}. Next, we will study the general features of the triviality surface.

A typical example is chosen as: $(h,\ \lambda) = (h_A,\ \lambda_A)$, $\phi = 0$, $\tilde{v}_3 = 0.7\, M_W$. Its triviality surface is computed, and B_{HH} versus $m_1^2\,(= m_2^2)$ is plotted in Fig.1. Fig.1 corresponds to a line on the triviality surface. Several universal features of Fig.1 are important. First, when m_1^2 is small, the soft SUSY-breaking terms are not important and therefore the determination of B_{HH} is insensitive to m_1^2. Second, the curve in Fig.1 ends at $m_1^2 \approx -M_W^2$ because the squared-mass matrix $[M_n^2]$ or $[M_c^2]$ will develop negative eigenvalues if m_1^2 becomes too negative. Together with (30), this observation indicates that m_1^2, m_2^2, $\tilde{m}_{12}^2$ are bounded from below. Since nothing interesting happens when $m_1^2 < 0$, we will assume $0 \le m_1 = m_2 < \tilde{m}_{12}$ from now on.

The last but most important universal feature of Fig.1 is: When m_1^2 is large, the linear relation $B_{HH} = \sqrt{2}\, m_1$ is a good approximation. This observation can be understood as follows. (13) implies that, on the triviality surface, h^2 and $\lambda^2 \to 0$ if $B_{HH}\,(= \Lambda_L) \to \infty$. The structure of $[M_n^2]$ also implies $B_{HH} \to \infty$ if $m_1 = m_2 \to \infty$. The above two observations lead to:

$$\text{On the triviality surface, } h^2 \text{ and } \lambda^2 \to 0 \text{ if } m_1 = m_2 \to \infty \tag{38}$$

Therefore, in the limit $m_1 = m_2 \to \infty$ on the triviality surface, $[M_n^2]$ and $[M_c^2]$ can be solved up to order $O(h^2)$ exactly:

$$\begin{aligned} \text{Eigenvalues of } [M_n^2] &= [2m_1^2 + O(h^2),\ 2m_1^2 + O(h^2),\ O(h^2),\ O(h^2),\ O(h^2),\ 0] \\ \text{Eigenvalues of } [M_c^2] &= [2m_1^2 + O(h^2),\ 2m_1^2 + O(h^2),\ 0,\ 0] \end{aligned} \tag{39}$$

The square roots of the seven non-zero eigenvalues in (39) are just the seven triviality upper bounds on the seven Higgs masses respectively, including the triviality upper bound on m_{LH}. (39) together with (38) explains why $B_{HH} = \sqrt{2}\, m_1$ is valid up to order $O(h^2)$ when m_1^2 is large. (39) also implies that there are exactly three light neutral Higgs bosons when m_1^2 is large. As for the lightest-Higgs mass m_{LH}, (39) indicates that the triviality upper bound on m_{LH} is of order $O(h)$, which is consistent with the upper bound $B_{LH}^{(BS)}$: $m_{LH} \le B_{LH}^{(BS)} = \frac{h}{\sqrt{2}} v$ in (37). Due to (38), the triviality upper bound on m_{LH} decreases to zero

monotonically as $m_1 = m_2 \to \infty$. According to the present experimental lower bound on the Higgs mass [26, 27], we require that the triviality upper bound on m_{LH} be larger than 1 M_W, and this requirement leads to an upper bound on m_1 $(= m_2)$ due to the fact that the triviality upper bound on m_{LH} decreases to zero as $m_1 = m_2 \to \infty$. An explicit realization of this idea is given in Fig.2, where $(h,\ \lambda) = (h_A,\ \lambda_A)$, $\phi = 0.6$, and $(\tilde{v}_3,\ m_1)$ is examined thoroughly. The enclosed region of Fig.2 is the allowed range of $\tilde{v}_3$ versus m_1. An interesting quantity B_{soft} can be defined in such a way that the allowed range of $\tilde{v}_3$ shrinks to a single point at $m_1 = B_{soft}$ and there is no solution for $m_1 > B_{soft}$. In Fig.2, $B_{soft} = 138\ M_W$. The meaning of B_{soft} is clear: For given ϕ, B_{soft} is the absolute upper bound on m_1 for $0 < \tilde{v}_3 \leq \frac{M_W}{2g_2}$. That is, B_{soft} is the upper bound on m_1 when $\tilde{v}_3 = \frac{M_W}{2g_2}$, and the upper bound on m_1 is smaller than B_{soft} when $\tilde{v}_3 < \frac{M_W}{2g_2}$. In fact, B_{soft} can be interpreted as the absolute upper bound (with ϕ fixed) on all the soft SUSY-breaking parameters $(m_1,\ m_2,\ \tilde{m}_{12})$ because $m_1^2 \approx \tilde{m}_{12}^2$ is true on the triviality surface when m_1^2 is large. The fact that $m_1^2 \approx \tilde{m}_{12}^2$ on the triviality surface when m_1^2 is large can be explained by the observations that $h^2 \propto (\tilde{m}_{12}^2 - m_1^2)$ and that h^2 is negligible when m_1^2 is large.

The dependence of B_{soft} on ϕ is displayed in Fig.3, where $(h_A,\ \lambda_A)$ and $(h_B,\ \lambda_B)$ have identical results. In Fig.3, it is also required that the triviality upper bound on m_{LH} be larger than 1 M_W. Fig.3 and Fig.2 form the complete picture of the triviality upper bound on the soft SUSY-breaking parameters. For example, $B_{soft} = 2380\ M_W$ at $\phi = 0$, and $B_{soft} = 84.6\ M_W$ at $\phi = \frac{\pi}{2}$. Furthermore, all the conclusions about B_{soft} can be re-interpreted as the absolute triviality upper bound on the heaviest-Higgs mass (with ϕ fixed) by means of $B_{HH} \simeq \sqrt{2}\, B_{soft}$. Therefore, Fig.3 also provides the complete picture of the absolute triviality upper bound on the heaviest-Higgs mass m_{HH}. The computations of Fig.3 are very sensitive to the experimental lower bound on the Higgs mass. Fig.3 is obtained by requiring that the triviality upper bound on m_{LH} be larger than 1 M_W, and $B_{soft} = 2380\ M_W$ is obtained when $\phi = 0$. However, $B_{soft} = 4.88\ M_W$ at $\phi = 0$ will be obtained if we require that the triviality upper bound on m_{LH} be larger than 2 M_W.

Inspired by Fig.2, we can define the absolute triviality lower bound B_N on $\tilde{v}_3$ for given ϕ. For example, $B_N = 0.7\ M_W$ in Fig.2. B_N gives a modest

measure of the constraint on $\tilde{v}_3$. Fig.4 displays the dependence of B_N on ϕ for $(h_A,\ \lambda_A)$ and $(h_B,\ \lambda_B)$. Besides, it is required that the triviality upper bound on m_{LH} be larger than 1 M_W. The dotted straight line corresponds to the absolute upper bound of 0.749 M_W on $\tilde{v}_3$, (28). For $\phi = 0$ (i.e., no CP-violation in the scalar sector), we have $0.24\, M_W \le \tilde{v}_3 \le 0.749\, M_W$. For $\phi = \frac{\pi}{2}$, $0.65\, M_W \le \tilde{v}_3 \le 0.749\, M_W$. Therefore, triviality is very helpful for a better understanding of $\tilde{v}_3$. In addition, if the experimental lower bound on the Higgs mass is raised in the future, all the bounds involved in Fig.3 and Fig.4 will become stronger, which implies a better understanding of the heaviest-Higgs mass, the VEV of the Higgs singlet, and the soft SUSY-breaking parameters. However, NMSSM is not consistent with an unlimited raise of the experimental lower bound on the Higgs mass. In Section 5, we will derive an absolute upper bound of 2.8 M_W on the lightest-Higgs mass.

Finally, let's check the assumptions of negligible f_t and g_i. With the help of [15], all the computations involved in Figures 1-4 do satisfy the assumption of negligible g_i. To check the assumption of negligible f_t, take the mass of top quark $m_t = 170$ GeV. Generally speaking, this assumption is reasonable when m_1^2 is small, but it needs modifications when m_1^2 is large because h^2 and λ^2 are small according to (38). As for Fig.4, the computation of B_N indicates that f_t is negligible. However, the computation of B_{soft} in Fig.3 indicates that f_t^2 is as important as h^2 and λ^2. To understand the effect of f_t^2 on B_{soft}, refer to (6)-(8). Because all the coefficients of f_t^2-terms in (6)-(8) are positive (assuming negligible g_i), the inclusion of f_t makes triviality even stronger. That is, the Landau pole t_L will be smaller if f_t is included. Qualitatively, it implies that B_{soft} should be smaller (i.e., a stronger upper bound) if f_t is included. In general, all the triviality bounds will become stronger if f_t is included. In other words, the results of Fig.3 should be regarded as a weak absolute upper bound on the soft SUSY-breaking parameters and the heaviest-Higgs mass.

5 Absolute Upper Bound on the Lightest-Higgs Mass

With the inclusion of the Higgs singlet in NMSSM, the tree-level upper bound on the lightest-Higgs mass of MSSM is no longer valid. Therefore, it is of considerable importance to study the triviality upper bound on the lightest-

Higgs mass in order to devise effective search strategies for the detection of Higgs particles. For given ϕ, we can define a new quantity B_{LH}, the absolute triviality upper bound B_{LH} on the lightest-Higgs mass, as the largest triviality upper bound on the lightest-Higgs mass with respect to all the possible values of $m_1\,(=m_2)$ and $\tilde{v}_3$. However, (39) implies that a search in the small-m_1^2 regime is enough, and the dependence of B_{LH} on ϕ is displayed in Fig.5, where line A and line B correspond to $(h_A,\ \lambda_A)$ and $(h_B,\ \lambda_B)$ respectively. It is verified that Fig.5 satisfies the assumptions of negligible f_t and g_i.

When $\phi = 0$ (i.e., no CP-violation in the scalar sector), the absolute upper bound $B_{LH} = 2.8\ M_W$ for line B. When $\phi = \frac{\pi}{2}$, $B_{LH} = 1.75\ M_W$ for line B. Therefore, the absolute triviality upper bound on the lightest-Higgs mass does lie outside the range of LEP.

6 Conclusion

With a complete study of the triviality surface, we are able to derive the triviality bounds on the heaviest-Higgs mass, the soft SUSY-breaking parameters, the VEV of the Higgs singlet in Section 4, and the absolute upper bound on the lightest-Higgs mass in Section 5. Essentially, all the triviality bounds are derived based on the observations (38) and (39), where the triviality upper bound on the lightest-Higgs mass decreases to zero as $m_1 = m_2 \to \infty$.

The particular choice of the soft SUSY-breaking potential V_{soft} in (2) can be viewed as an unsatisfactory feature of the present formulation. However, the triviality bounds derived in Sections 4 and 5 persist even if a more general V_{soft} is considered. We begin the argument of the above statement with the most general V_{soft} [10, 21]:

$$\begin{aligned} V_{soft} \;\; = \;\; & m_1^2 {\Phi^\dagger}_1\Phi_1 + m_2^2{\Phi^\dagger}_2\Phi_2 - m_{12}^2{\Phi^\dagger}_1\Phi_2 - {m^*}_{12}^2{\Phi^\dagger}_2\Phi_1 \\ & + m_4^2 N^* N + m_5^2 N^2 + {m^*}_5^2 N^{*2} \\ & + h\, m_3 (A_1 \Phi_1^\dagger \Phi_2 N + {A^*}_1 \Phi_2^\dagger \Phi_1 N^*) \\ & + \frac{1}{3}\,\lambda\, m_3 (A_2 N^3 + {A^*}_2 N^{*3}) \end{aligned} \tag{40}$$

Now, the relevant parameter space consists of:

$$(h,\ \lambda,\ v_1,\ v_2,\ v_3,\ m_1^2,\ m_2^2,\ m_3,\ m_4^2,\ m_5^2,\ m_{12}^2,\ A_1,\ A_2) \tag{41}$$

plus three complex vacuum constraints on the parameters derived from the minimization of $V' = V + V_{soft}$ and (19). Choosing $\tan\beta = 1$, we have $m_1^2 = m_2^2$ from the minimization of V' again. In the limit of large $\Lambda_L\,(= B_{HH})$, the Landau pole (13) always implies:

$$h^2 \ \rightarrow\ 0 \ \text{ if } \ \Lambda_L\,(= B_{HH})\ \rightarrow\ \infty \tag{42}$$

In the large-m_{HH} limit (e.g., in the large-m_1 limit) on the triviality surface, (42) implies that $[M_n^2]$ and $[M_c^2]$ can be solved up to order $O(h)$ exactly:

$$\begin{aligned}
\text{Eigenvalues of } [M_n^2] &= [2m_1^2 + O(h),\ 2m_1^2 + O(h),\ m_{(+)} + O(h),\ m_{(-)} + O(h),\ O(h),\ 0] \\
\text{Eigenvalues of } [M_c^2] &= [2m_1^2 + O(h),\ 2m_1^2 + O(h),\ 0,\ 0] \\
m_{(\pm)} &= m_4^2 + 4\lambda^2|v_3|^2 \pm 2|m_5^2 + \lambda m_3 v_3 A_2 + \lambda^2 v_3^{*2}|
\end{aligned} \tag{43}$$

With the most general V_{soft} (40), there is, in general, exactly one Higgs boson staying light in the large-m_{HH} limit. According to (42) and (43), the triviality upper bound on m_{LH} is of order $O(h^{\frac{1}{2}})$, and decreases to zero as $m_{HH} \rightarrow \infty$. This observation implies that the analyses of Sections 4 and 5 still apply to the most general $V' = V + V_{soft}$. That is, the triviality bounds on the heaviest-Higgs mass, the lightest-Higgs mass, the VEV of the Higgs singlet, and the soft SUSY-breaking parameters will not be lost even if the largest parameter space of NMSSM is considered.

Finally, two aspects of the present computations can be improved. First, the computation of the effective potential in this paper is performed only at tree level. Because the contributions of the top and stop loops are important, it is necessary for future works to include one-loop contributions. Second, $\tan\beta = 1$ is chosen in this paper for the sake of simplicity. However, this particular choice has no physical motivation. Therefore, choices different from $\tan\beta = 1$ should be considered and the discussion of the $\tan\beta$-dependence may be a point of interest in future works.

Acknowledgement

I would like to thank Professor Mary K. Gaillard for her support and nice advice. I also thank Dr. H.-C. Cheng for discussions about MSSM. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY90-21139.

References

[1] M.A.B. Bég, in *Higgs Particles*, edited by A. Ali (Plenum Press, New York, 1990), pp.7-38.

[2] *The Higgs Hunter's Guide* by J.F. Gunion et al. (Addison-Wesley, MA, 1990), pp.17-154.

[3] D.J.E. Callaway, Phys. Rep. 167, 241 (1988).

[4] R. Dashen and H. Neuberger, Phys. Rev. Lett. 50, 1897 (1983).

[5] M. Lindner, Z. Phys. C 31, 295 (1986).

[6] B. Grzadkowski and M. Linder, Phys. Lett. 178B, 81 (1986).

[7] J. Kuti, L. Lin and Y. Shen, Phys. Rev. Lett. 61, 678 (1988).

[8] U.M. Heller et al., Nucl. Phys. B405, 555 (1993).

[9] E. Witten, Nucl. Phys. B188, 513 (1981).

[10] *The Higgs Hunter's Guide* by J.F. Gunion et al. (Addison-Wesley, MA, 1990).

[11] J.P. Derendinger and C.A. Savoy, Nucl. Phys. B237, 307 (1984).

[12] J. Ellis et al., Phys. Rev. D39, 844 (1989).

[13] H.P. Nilles, Phys. Rep. 110, 1 (1984).

[14] E. Cohen et al., Phys. Lett. 165B, 76 (1985).

[15] J.P. Derendinger et al., Nucl. Phys. B267, 365 (1986).

[16] H.E. Haber and R. Hempfling, Phys. Rev. Lett. 66, 1815 (1991).

[17] W.T.A. ter Veldhuis, PURD-TH-92-11.

[18] T. Moroi and Y. Okada, Phys. Lett. 295B, 73 (1992).

[19] P. Binétruy and C.A. Savoy, Phys. Lett. 277B, 453 (1992).

[20] T. Elliott et al., Phys. Rev. D49, 2435 (1994).
J.R. Espinosa and M. Quirós, Phys. Lett. 302B, 51 (1993).
P.N. Pandita, Phys. Lett. 318B, 338 (1993).
L. Durand and J.L. Lopez, Phys. Lett. 217B, 463 (1989).

[21] K.S. Babu and E. Ma, Phys. Rev. D33, 3334 (1986).

[22] W.J. Marciano and A. Sirlin, in *The Second Workshop on Grand Unification*, edited by J.P. Leveille et al. (Birkaüser, Boston, 1981), pp.151-162.

[23] *Advanced Engineering Mathematics* by P.V. O'Neil (Wadsworth Publishing, Taiwan, 1983).

[24] P. Hasenfratz and J. Nager, Z. Phys. C 37, 477 (1988).

[25] A. Hasenfratz and T. Neuhaus, Nucl. Phys. B297, 205 (1988).

[26] J.L. Lopez et al., Phys. Lett. 306B, 73 (1993).

[27] *Research Directions For The Decade* edited by E.L. Berger (World Scientific, Singapore, 1990).

FIGURE CAPTIONS

Fig.1: A plot of the triviality upper bound B_{HH} (M_W) on the heaviest-Higgs mass versus m_1^2 (M_W^2) for $\phi = 0$, $\tilde{v}_3 = 0.7\, M_W$, and $(h,\ \lambda) = (h_A,\ \lambda_A)$, where the unit $M_W = 80$ GeV.

Fig.2: A plot of the allowed range (the enclosed region) of $\tilde{v}_3$ (M_W) versus m_1 (M_W) for $\phi = 0.6$, $(h,\ \lambda) = (h_A,\ \lambda_A)$, where the unit $M_W = 80$ GeV. The allowed range of $\tilde{v}_3$ shrinks to a point at m_1=138 GeV$\equiv B_{soft}$.

Fig.3: The plot of the absolute triviality upper bound B_{soft} (M_W) versus ϕ (the unit $M_W = 80$ GeV), where the two solutions $(h_A,\ \lambda_A)$ and $(h_B,\ \lambda_B)$ have identical results. B_{soft} is periodic in ϕ with period π.

Fig.4: The plot of B_N (M_W), the absolute triviality lower bound on $\tilde{v}_3$, versus ϕ (the unit $M_W = 80$ GeV), where the dashed line corresponds to $(h_A,\ \lambda_A)$ and the solid line corresponds to $(h_B,\ \lambda_B)$. B_N is periodic in ϕ with period π. The dotted line corresponds to the absolute upper bound of $\frac{M_W}{2g_2}$ on $\tilde{v}_3$.

Fig.5: The plot of B_{LH} (M_W), the absolute triviality upper bound on the lightest-Higgs mass versus ϕ (the unit $M_W = 80$ GeV), where the dashed line corresponds to $(h_A,\ \lambda_A)$ and the solid line corresponds to $(h_B,\ \lambda_B)$. B_{LH} is periodic in ϕ with period π.